CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Addition: meanings, representations, mental calculation and algorithms 1 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on addition: meanings, representations, mental calculation and algorithms.
2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.
3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Study in deep the pedagogical content of arithmetic;
2. Planning in details the content to be developed, with focus on *addition: meanings, representations, mental calculation and algorithms*;
3. Developing and writing sequences of activities for autonomous study;
4. Developing worksheets to support teachers' work;
5. Translation and adaptation of NCETM's slides
6. Participate on a weekly seminar to present and discuss the material developed by the group;
7. Final polishing of educational resources bringing together the parts developed by different members of the team.

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X				
2		X	X			
3			X	X	X	X
4			X	X	X	X
5				X	X	X
6	X	X	X	X	X	X
7					X	X

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Addition: meanings, representations, mental calculation and algorithms 1 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

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2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on *addition: meanings, representations, mental calculation and algorithms*.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Developing and writing sequences of activities for autonomous study;
2. Developing worksheets to support teachers' work;
3. Translation and adaptation of NCETM's slides
4. Participate on a weekly seminar to present and discuss the material developed by the group;
5. Final polishing of educational resources bringing together the parts developed by different members of the team;
6. Evaluation of the material, considering actual use in classroom;

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X	X	X		
2	X	X	X	X		
3	X	X				
4	X	X	X	X	X	X
5			X	X	X	X
6	X	X	X	X		

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CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Addition: meanings, representations, mental calculation and algorithms 1 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

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2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on addition: meanings, representations, mental calculation and algorithms.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Study in deep the pedagogical content of arithmetic;
2. Planning in details the content to be developed, with focus on *addition: meanings, representations, mental calculation and algorithms*;
3. Developing and writing sequences of activities for autonomous study;
4. Developing worksheets to support teachers' work;
5. Translation and adaptation of NCETM's slides
6. Participate on a weekly seminar to present and discuss the material developed by the group;
7. Final polishing of educational resources bringing together the parts developed by different members of the team.

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X				
2		X	X			
3			X	X	X	X
4			X	X	X	X
5				X	X	X
6	X	X	X	X	X	X
7					X	X

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CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Addition: meanings, representations, mental calculation and algorithms 2 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on *addition: meanings, representations, mental calculation and algorithms*.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Developing and writing sequences of activities for autonomous study;
2. Developing worksheets to support teachers' work;
3. Translation and adaptation of NCETM's slides
4. Participate on a weekly seminar to present and discuss the material developed by the group;
5. Final polishing of educational resources bringing together the parts developed by different members of the team;
6. Evaluation of the material, considering actual use in classroom;

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X	X	X		
2	X	X	X	X		
3	X	X				
4	X	X	X	X	X	X
5			X	X	X	X
6	X	X	X	X		

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

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CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Division: meanings, representations, mental calculation and algorithms 1 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

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The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on division: meanings, representations, mental calculation and algorithms.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Study in deep the pedagogical content of arithmetic;

2. Planning in details the content to be developed, with focus on *division: meanings, representations, mental calculation and algorithms*;

3. Developing and writing sequences of activities for autonomous study;

4. Developing worksheets to support teachers' work;

5. Translation and adaptation of NCETM's slides

6. Participate on a weekly seminar to present and discuss the material developed by the group;

7. Final polishing of educational resources bringing together the parts developed by different members of the team.

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X				
2		X	X			
3			X	X	X	X
4			X	X	X	X
5				X	X	X
6	X	X	X	X	X	X
7					X	X

References

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CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Division: meanings, representations, mental calculation and algorithms 1 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

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The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on *division: meanings, representations, mental calculation and algorithms*.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Developing and writing sequences of activities for autonomous study;
2. Developing worksheets to support teachers' work;
3. Translation and adaptation of NCETM's slides
4. Participate on a weekly seminar to present and discuss the material developed by the group;
5. Final polishing of educational resources bringing together the parts developed by different members of the team;
6. Evaluation of the material, considering actual use in classroom;

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X	X	X		
2	X	X	X	X		
3	X	X				
4	X	X	X	X	X	X
5			X	X	X	X
6	X	X	X	X		

References

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CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Division: meanings, representations, mental calculation and algorithms 2 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

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2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Study in deep the pedagogical content of arithmetic;
2. Planning in details the content to be developed, with focus on *division: meanings, representations, mental calculation and algorithms*;
3. Developing and writing sequences of activities for autonomous study;
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5. Translation and adaptation of NCETM's slides
6. Participate on a weekly seminar to present and discuss the material developed by the group;
7. Final polishing of educational resources bringing together the parts developed by different members of the team.

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X				
2		X	X			
3			X	X	X	X
4			X	X	X	X
5				X	X	X
6	X	X	X	X	X	X
7					X	X

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CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Division: meanings, representations, mental calculation and algorithms 2 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

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5. Final polishing of educational resources bringing together the parts developed by different members of the team;
6. Evaluation of the material, considering actual use in classroom;

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X	X	X		
2	X	X	X	X		
3	X	X				
4	X	X	X	X	X	X
5			X	X	X	X
6	X	X	X	X		

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CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Fractions: meanings, representations, and operations 1 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

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2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on fractions: meanings, representations, and operations.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Study in deep the pedagogical content of arithmetic;
2. Planning in details the content to be developed, with focus on *fractions, its meanings, representations, and operation*;
3. Developing and writing sequences of activities for autonomous study;
4. Developing worksheets to support teachers' work;
5. Translation and adaptation of NCETM's slides
6. Participate on a weekly seminar to present and discuss the material developed by the group;
7. Final polishing of educational resources bringing together the parts developed by different members of the team.

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Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X				
2		X	X			
3			X	X	X	X
4			X	X	X	X
5				X	X	X
6	X	X	X	X	X	X
7					X	X

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CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Fractions: meanings, representations, and operations 1 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

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1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on *fractions, its meanings, representations and operation*.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Developing and writing sequences of activities for autonomous study;
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6. Evaluation of the material, considering actual use in classroom;

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X	X	X		
2	X	X	X	X		
3	X	X				
4	X	X	X	X	X	X
5			X	X	X	X
6	X	X	X	X		

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CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Fractions: meanings, representations, and operations 2 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

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2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on fractions: meanings, representations, and operations.
2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.
3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Study in deep the pedagogical content of arithmetic;
2. Planning in details the content to be developed, with focus on *fractions, its meanings, representations, and operation*;
3. Developing and writing sequences of activities for autonomous study;
4. Developing worksheets to support teachers' work;
5. Translation and adaptation of NCETM's slides
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3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X				
2		X	X			
3			X	X	X	X
4			X	X	X	X
5				X	X	X
6	X	X	X	X	X	X
7					X	X

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CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Fractions: meanings, representations, and operations 2 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

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2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on *fractions, its meanings, representations and operation*.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

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3 Work Plan

3.1 Activities

1. Developing and writing sequences of activities for autonomous study;
2. Developing worksheets to support teachers' work;
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4. Participate on a weekly seminar to present and discuss the material developed by the group;
5. Final polishing of educational resources bringing together the parts developed by different members of the team;
6. Evaluation of the material, considering actual use in classroom;

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X	X	X		
2	X	X	X	X		
3	X	X				
4	X	X	X	X	X	X
5			X	X	X	X
6	X	X	X	X		

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Fractions: meanings, representations, and operations 3 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on fractions: meanings, representations, and operations.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Study in deep the pedagogical content of arithmetic;
2. Planning in details the content to be developed, with focus on *fractions, its meanings, representations, and operation*;
3. Developing and writing sequences of activities for autonomous study;
4. Developing worksheets to support teachers' work;
5. Translation and adaptation of NCETM's slides
6. Participate on a weekly seminar to present and discuss the material developed by the group;
7. Final polishing of educational resources bringing together the parts developed by different members of the team.

3.2 Schedule

Activity \Months	**1 - 2**	**3 - 4**	**5 - 6**	**7 - 8**	**9 - 10**	**11 - 12**
1	X	X				
2		X	X			
3			X	X	X	X
4			X	X	X	X
5				X	X	X
6	X	X	X	X	X	X
7					X	X

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Fractions: meanings, representations, and operations 3 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on *fractions, its meanings, representations and operation*.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Developing and writing sequences of activities for autonomous study;
2. Developing worksheets to support teachers' work;
3. Translation and adaptation of NCETM's slides
4. Participate on a weekly seminar to present and discuss the material developed by the group;
5. Final polishing of educational resources bringing together the parts developed by different members of the team;
6. Evaluation of the material, considering actual use in classroom;

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X	X	X		
2	X	X	X	X		
3	X	X				
4	X	X	X	X	X	X
5			X	X	X	X
6	X	X	X	X		

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Multiplication: meanings, representations, mental calculation and algorithms 1 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on multiplication: meanings, representations, mental calculation and algorithms.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Study in deep the pedagogical content of arithmetic;

2. Planning in details the content to be developed, with focus on *multiplication: meanings, representations, mental calculation and algorithms*;

3. Developing and writing sequences of activities for autonomous study;

4. Developing worksheets to support teachers' work;

5. Translation and adaptation of NCETM's slides

6. Participate on a weekly seminar to present and discuss the material developed by the group;

7. Final polishing of educational resources bringing together the parts developed by different members of the team.

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X				
2		X	X			
3			X	X	X	X
4			X	X	X	X
5				X	X	X
6	X	X	X	X	X	X
7					X	X

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Multiplication: meanings, representations, mental calculation and algorithms 1 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on *multiplication: meanings, representations, mental calculation and algorithms*.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Developing and writing sequences of activities for autonomous study;

2. Developing worksheets to support teachers' work;

3. Translation and adaptation of NCETM's slides

4. Participate on a weekly seminar to present and discuss the material developed by the group;

5. Final polishing of educational resources bringing together the parts developed by different members of the team;

6. Evaluation of the material, considering actual use in classroom;

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X	X	X		
2	X	X	X	X		
3	X	X				
4	X	X	X	X	X	X
5			X	X	X	X
6	X	X	X	X		

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Multiplication: meanings, representations, mental calculation and algorithms 2 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on multiplication: meanings, representations, mental calculation and algorithms.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Study in deep the pedagogical content of arithmetic;

2. Planning in details the content to be developed, with focus on *multiplication: meanings, representations, mental calculation and algorithms*;

3. Developing and writing sequences of activities for autonomous study;

4. Developing worksheets to support teachers' work;

5. Translation and adaptation of NCETM's slides

6. Participate on a weekly seminar to present and discuss the material developed by the group;

7. Final polishing of educational resources bringing together the parts developed by different members of the team.

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X				
2		X	X			
3			X	X	X	X
4			X	X	X	X
5				X	X	X
6	X	X	X	X	X	X
7					X	X

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Multiplication: meanings, representations, mental calculation and algorithms 2 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on *multiplication: meanings, representations, mental calculation and algorithms*.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Developing and writing sequences of activities for autonomous study;
2. Developing worksheets to support teachers' work;
3. Translation and adaptation of NCETM's slides
4. Participate on a weekly seminar to present and discuss the material developed by the group;
5. Final polishing of educational resources bringing together the parts developed by different members of the team;
6. Evaluation of the material, considering actual use in classroom;

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X	X	X		
2	X	X	X	X		
3	X	X				
4	X	X	X	X	X	X
5			X	X	X	X
6	X	X	X	X		

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Number sense and positional number system 1 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on number sense, counting and positional number system.
2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.
3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Study in deep the pedagogical content of arithmetic;
2. Planning in details the content to be developed, with focus on *counting and positional number system*;
3. Developing and writing sequences of activities for autonomous study;
4. Developing worksheets to support teachers' work;
5. Translation and adaptation of NCETM's slides
6. Participate on a weekly seminar to present and discuss the material developed by the group;
7. Final polishing of educational resources bringing together the parts developed by different members of the team.

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X				
2		X	X			
3			X	X	X	X
4			X	X	X	X
5				X	X	X
6	X	X	X	X	X	X
7					X	X

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Number sense and positional number system 1 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on number sense, counting and positional number system.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Developing and writing sequences of activities for autonomous study;
2. Developing worksheets to support teachers' work;
3. Translation and adaptation of NCETM's slides
4. Participate on a weekly seminar to present and discuss the material developed by the group;
5. Final polishing of educational resources bringing together the parts developed by different members of the team;
6. Evaluation of the material, considering actual use in classroom;

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X	X	X		
2	X	X	X	X		
3	X	X				
4	X	X	X	X	X	X
5			X	X	X	X
6	X	X	X	X		

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Number sense and positional number system 2 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on number sense, counting and positional number system.
2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.
3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Study in deep the pedagogical content of arithmetic;
2. Planning in details the content to be developed, with focus on *counting and positional number system*;
3. Developing and writing sequences of activities for autonomous study;
4. Developing worksheets to support teachers' work;
5. Translation and adaptation of NCETM's slides
6. Participate on a weekly seminar to present and discuss the material developed by the group;
7. Final polishing of educational resources bringing together the parts developed by different members of the team.

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X				
2		X	X			
3			X	X	X	X
4			X	X	X	X
5				X	X	X
6	X	X	X	X	X	X
7					X	X

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Number sense and positional number system 2 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on number sense, counting and positional number system.
2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.
3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Developing and writing sequences of activities for autonomous study;
2. Developing worksheets to support teachers' work;
3. Translation and adaptation of NCETM's slides
4. Participate on a weekly seminar to present and discuss the material developed by the group;
5. Final polishing of educational resources bringing together the parts developed by different members of the team;
6. Evaluation of the material, considering actual use in classroom;

3.2 Schedule

Activity \Months	**1 - 2**	**3 - 4**	**5 - 6**	**7 - 8**	**9 - 10**	**11 - 12**
1	X	X	X	X		
2	X	X	X	X		
3	X	X				
4	X	X	X	X	X	X
5			X	X	X	X
6	X	X	X	X		

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Subtraction: meanings, representations, mental calculation and algorithms 1 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on subtraction: meanings, representations, mental calculation and algorithms.
2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.
3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Study in deep the pedagogical content of arithmetic;
2. Planning in details the content to be developed, with focus on *subtraction: meanings, representations, mental calculation and algorithms*;
3. Developing and writing sequences of activities for autonomous study;
4. Developing worksheets to support teachers' work;
5. Translation and adaptation of NCETM's slides
6. Participate on a weekly seminar to present and discuss the material developed by the group;
7. Final polishing of educational resources bringing together the parts developed by different members of the team.

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X				
2		X	X			
3			X	X	X	X
4			X	X	X	X
5				X	X	X
6	X	X	X	X	X	X
7					X	X

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Subtraction: meanings, representations, mental calculation and algorithms 1 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on *subtraction: meanings, representations, mental calculation and algorithms*.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Developing and writing sequences of activities for autonomous study;
2. Developing worksheets to support teachers' work;
3. Translation and adaptation of NCETM's slides
4. Participate on a weekly seminar to present and discuss the material developed by the group;
5. Final polishing of educational resources bringing together the parts developed by different members of the team;
6. Evaluation of the material, considering actual use in classroom;

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X	X	X		
2	X	X	X	X		
3	X	X				
4	X	X	X	X	X	X
5			X	X	X	X
6	X	X	X	X		

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Subtraction: meanings, representations, mental calculation and algorithms 2 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on subtraction: meanings, representations, mental calculation and algorithms.
2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.
3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Study in deep the pedagogical content of arithmetic;
2. Planning in details the content to be developed, with focus on *subtraction: meanings, representations, mental calculation and algorithms*;
3. Developing and writing sequences of activities for autonomous study;
4. Developing worksheets to support teachers' work;
5. Translation and adaptation of NCETM's slides
6. Participate on a weekly seminar to present and discuss the material developed by the group;
7. Final polishing of educational resources bringing together the parts developed by different members of the team.

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X				
2		X	X			
3			X	X	X	X
4			X	X	X	X
5				X	X	X
6	X	X	X	X	X	X
7					X	X

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Arithmetic in Basic Education
Subtraction: meanings, representations, mental calculation and algorithms 2 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

Deep understanding and proficiency in arithmetic is a key point in all future school program. Few Brazilian students has an appropriate knowledge on this subject and both federal and state governments find the need to support remedial policies or closer follow up of students development. In 2021, according to the *National System for the Evaluation of Basic Education* - SAEB results of 2021, only 36% of fifth grade students attained proficiency results.

Deep understanding of arithmetic is a key to future development in mathematics (see [1]) and knowledge of fractions can even predict future mastering in Algebra (Sowder e Wearne, 2006; Wearne e Kouba, 2000; apud [2]).

The context of this project is the development of a variety of tools to help tackle this problem and bridge the gap in elementary mathematics education. Those instruments could be used, in whole or in part, by teachers, schools or educational systems.

2 Objectives

1. Develop the content for a software and app devoted for autonomous study of structure of arithmetic, with focus on *subtraction: meanings, representations, mental calculation and algorithms*.

2. Adapt and translate the slides developed by the *National Centre for Excellence in the Teaching of Mathematics* (NCETM) with focus on the first spine.

3. Develop the content of worksheets to support teachers' and tutors' work with small groups of students.

3 Work Plan

3.1 Activities

1. Developing and writing sequences of activities for autonomous study;
2. Developing worksheets to support teachers' work;
3. Translation and adaptation of NCETM's slides
4. Participate on a weekly seminar to present and discuss the material developed by the group;
5. Final polishing of educational resources bringing together the parts developed by different members of the team;
6. Evaluation of the material, considering actual use in classroom;

3.2 Schedule

Activity \Months	1 - 2	3 - 4	5 - 6	7 - 8	9 - 10	11 - 12
1	X	X	X	X		
2	X	X	X	X		
3	X	X				
4	X	X	X	X	X	X
5			X	X	X	X
6	X	X	X	X		

References

[1] MA, Liping. Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States. Routledge, 2010. 1

[2] VAN DE WALLE, John A. Elementary and Middle School Mathematics: Teaching Developmentally. New Jersey: Pearson, 1990. 1

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Algebra 2 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

This scholarship is intended for deaf students in the STEM field. He will be part of a group that will record in Libras the answers to more than 1400 ENEM mathematics questions. He will be one of four students focusing on number theory and basic arithmetic operations. This program is a consequence and continuation of a pilot project run at Unicamp in 2023.

2 Objectives

1. Promote social inclusion of deaf people in Brazil

2. Develop the vocabulary of mathematics in Brazilian Sign Language - Libras

3. Create the first content of a website that will be the main reference for studying mathematics in Libras.

4. Develop a community of deaf people engaged in learning mathematics

This will be achieved by recording in Libras the answer to each of the mathematics questions of ENEM - Brazilian National High School Exam. There are more than 1400 questions considering all the editions since 2009.

3 Work Plan

3.1 Activities

1. Start up seminar: two weeks at Unicamp, introduction to development tools and the use of the tools for recording classes. First videos productions and analysis.
2. Weekly recording of video.
3. Weekly meeting: 4 hours of weekly discussion of future subjects and analysis and correction of recorded videos
4. Summer seminar: two weeks at Unicamp, studying the needed mathematics of the upcoming year.
5. Winter seminar: one week at Unicamp, intensive work on video and studying mathematics

3.2 Schedule

Activity	Months 1 to 3	4 to 6	7 to 9	10 to 12
Start up seminar	X			
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X
Summer seminar			X	X
Winter seminar		X		

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Algebra 2 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

This scholarship is intended for deaf students in the STEM field. He will be part of a group that will record in Libras the answers to more than 1400 ENEM mathematics questions. He will be one of four students focusing on number theory and basic arithmetic operations. This program is a consequence and continuation of a pilot project run at Unicamp in 2023.

2 Objectives

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2. Develop the vocabulary of mathematics in Brazilian Sign Language - Libras
3. Create the first content of a website that will be the main reference for studying mathematics in Libras.
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This will be achieved by recording in Libras the answer to each of the mathematics questions of ENEM - Brazilian National High School Exam. There are more than 1400 questions considering all the editions since 2009.

3 Work Plan

3.1 Activities

1. Start up seminar: two weeks at Unicamp, introduction to development tools and the use of the tools for recording classes. First videos productions and analysis.
2. Weekly recording of video.
3. Weekly meeting: 4 hours of weekly discussion of future subjects and analysis and correction of recorded videos
4. Summer seminar: two weeks at Unicamp, studying the needed mathematics of the upcoming year.
5. Winter seminar: one week at Unicamp, intensive work on video and studying mathematics

3.2 Schedule

Activity	Months 1’ to 3’	4’ to 6’	7’ to 9’	10’ to 12’
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Algebra 3 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

This scholarship is intended for deaf students in the STEM field. He will be part of a group that will record in Libras the answers to more than 1400 ENEM mathematics questions. He will be one of four students focusing on number theory and basic arithmetic operations. This program is a consequence and continuation of a pilot project run at Unicamp in 2023.

2 Objectives

1. Promote social inclusion of deaf people in Brazil

2. Develop the vocabulary of mathematics in Brazilian Sign Language - Libras

3. Create the first content of a website that will be the main reference for studying mathematics in Libras.

4. Develop a community of deaf people engaged in learning mathematics

This will be achieved by recording in Libras the answer to each of the mathematics questions of ENEM - Brazilian National High School Exam. There are more than 1400 questions considering all the editions since 2009.

3 Work Plan

3.1 Activities

1. Start up seminar: two weeks at Unicamp, introduction to development tools and the use of the tools for recording classes. First videos productions and analysis.
2. Weekly recording of video.
3. Weekly meeting: 4 hours of weekly discussion of future subjects and analysis and correction of recorded videos
4. Summer seminar: two weeks at Unicamp, studying the needed mathematics of the upcoming year.
5. Winter seminar: one week at Unicamp, intensive work on video and studying mathematics

3.2 Schedule

Activity	Months 1 to 3	4 to 6	7 to 9	10 to 12
Start up seminar	X			
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X
Summer seminar			X	X
Winter seminar		X		

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Algebra 3 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

This scholarship is intended for deaf students in the STEM field. He will be part of a group that will record in Libras the answers to more than 1400 ENEM mathematics questions. He will be one of four students focusing on number theory and basic arithmetic operations. This program is a consequence and continuation of a pilot project run at Unicamp in 2023.

2 Objectives

1. Promote social inclusion of deaf people in Brazil
2. Develop the vocabulary of mathematics in Brazilian Sign Language - Libras
3. Create the first content of a website that will be the main reference for studying mathematics in Libras.
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This will be achieved by recording in Libras the answer to each of the mathematics questions of ENEM - Brazilian National High School Exam. There are more than 1400 questions considering all the editions since 2009.

3 Work Plan

3.1 Activities

1. Start up seminar: two weeks at Unicamp, introduction to development tools and the use of the tools for recording classes. First videos productions and analysis.
2. Weekly recording of video.
3. Weekly meeting: 4 hours of weekly discussion of future subjects and analysis and correction of recorded videos
4. Summer seminar: two weeks at Unicamp, studying the needed mathematics of the upcoming year.
5. Winter seminar: one week at Unicamp, intensive work on video and studying mathematics

3.2 Schedule

Activity	Months 1' to 3'	4' to 6'	7' to 9'	10' to 12'
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Algebra 4 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

This scholarship is intended for deaf students in the STEM field. He will be part of a group that will record in Libras the answers to more than 1400 ENEM mathematics questions. He will be one of four students focusing on number theory and basic arithmetic operations. This program is a consequence and continuation of a pilot project run at Unicamp in 2023.

2 Objectives

1. Promote social inclusion of deaf people in Brazil

2. Develop the vocabulary of mathematics in Brazilian Sign Language - Libras

3. Create the first content of a website that will be the main reference for studying mathematics in Libras.

4. Develop a community of deaf people engaged in learning mathematics

This will be achieved by recording in Libras the answer to each of the mathematics questions of ENEM - Brazilian National High School Exam. There are more than 1400 questions considering all the editions since 2009.

3 Work Plan

3.1 Activities

1. Start up seminar: two weeks at Unicamp, introduction to development tools and the use of the tools for recording classes. First videos productions and analysis.
2. Weekly recording of video.
3. Weekly meeting: 4 hours of weekly discussion of future subjects and analysis and correction of recorded videos
4. Summer seminar: two weeks at Unicamp, studying the needed mathematics of the upcoming year.
5. Winter seminar: one week at Unicamp, intensive work on video and studying mathematics

3.2 Schedule

Activity	Months 1 to 3	4 to 6	7 to 9	10 to 12
Start up seminar	X			
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X
Summer seminar			X	X
Winter seminar		X		

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Algebra 4 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

This scholarship is intended for deaf students in the STEM field. He will be part of a group that will record in Libras the answers to more than 1400 ENEM mathematics questions. He will be one of four students focusing on number theory and basic arithmetic operations. This program is a consequence and continuation of a pilot project run at Unicamp in 2023.

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1. Promote social inclusion of deaf people in Brazil

2. Develop the vocabulary of mathematics in Brazilian Sign Language - Libras

3. Create the first content of a website that will be the main reference for studying mathematics in Libras.

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3 Work Plan

3.1 Activities

1. Start up seminar: two weeks at Unicamp, introduction to development tools and the use of the tools for recording classes. First videos productions and analysis.
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3. Weekly meeting: 4 hours of weekly discussion of future subjects and analysis and correction of recorded videos
4. Summer seminar: two weeks at Unicamp, studying the needed mathematics of the upcoming year.
5. Winter seminar: one week at Unicamp, intensive work on video and studying mathematics

3.2 Schedule

Activity	Months 1' to 3'	4' to 6'	7' to 9'	10' to 12'
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Statistic and Probability 1 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

This scholarship is intended for deaf students in the STEM field. He will be part of a group that will record in Libras the answers to more than 1400 ENEM mathematics questions. He will be one of four students focusing on number theory and basic arithmetic operations. This program is a consequence and continuation of a pilot project run at Unicamp in 2023.

2 Objectives

1. Promote social inclusion of deaf people in Brazil

2. Develop the vocabulary of mathematics in Brazilian Sign Language - Libras

3. Create the first content of a website that will be the main reference for studying mathematics in Libras.

4. Develop a community of deaf people engaged in learning mathematics

This will be achieved by recording in Libras the answer to each of the mathematics questions of ENEM - Brazilian National High School Exam. There are more than 1400 questions considering all the editions since 2009.

3 Work Plan

3.1 Activities

1. Start up seminar: two weeks at Unicamp, introduction to development tools and the use of the tools for recording classes. First videos productions and analysis.
2. Weekly recording of video.
3. Weekly meeting: 4 hours of weekly discussion of future subjects and analysis and correction of recorded videos
4. Summer seminar: two weeks at Unicamp, studying the needed mathematics of the upcoming year.
5. Winter seminar: one week at Unicamp, intensive work on video and studying mathematics

3.2 Schedule

Activity	Months 1 to 3	4 to 6	7 to 9	10 to 12
Start up seminar	X			
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X
Summer seminar			X	X
Winter seminar		X		

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Statistic and Probability 1 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

This scholarship is intended for deaf students in the STEM field. He will be part of a group that will record in Libras the answers to more than 1400 ENEM mathematics questions. He will be one of four students focusing on number theory and basic arithmetic operations. This program is a consequence and continuation of a pilot project run at Unicamp in 2023.

2 Objectives

1. Promote social inclusion of deaf people in Brazil

2. Develop the vocabulary of mathematics in Brazilian Sign Language - Libras

3. Create the first content of a website that will be the main reference for studying mathematics in Libras.

4. Develop a community of deaf people engaged in learning mathematics

This will be achieved by recording in Libras the answer to each of the mathematics questions of ENEM - Brazilian National High School Exam. There are more than 1400 questions considering all the editions since 2009.

3 Work Plan

3.1 Activities

1. Start up seminar: two weeks at Unicamp, introduction to development tools and the use of the tools for recording classes. First videos productions and analysis.
2. Weekly recording of video.
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4. Summer seminar: two weeks at Unicamp, studying the needed mathematics of the upcoming year.
5. Winter seminar: one week at Unicamp, intensive work on video and studying mathematics

3.2 Schedule

Activity	Months 1' to 3'	4' to 6'	7' to 9'	10' to 12'
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Statistic and Probability 2 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

This scholarship is intended for deaf students in the STEM field. He will be part of a group that will record in Libras the answers to more than 1400 ENEM mathematics questions. He will be one of four students focusing on number theory and basic arithmetic operations. This program is a consequence and continuation of a pilot project run at Unicamp in 2023.

2 Objectives

1. Promote social inclusion of deaf people in Brazil

2. Develop the vocabulary of mathematics in Brazilian Sign Language - Libras

3. Create the first content of a website that will be the main reference for studying mathematics in Libras.

4. Develop a community of deaf people engaged in learning mathematics

This will be achieved by recording in Libras the answer to each of the mathematics questions of ENEM - Brazilian National High School Exam. There are more than 1400 questions considering all the editions since 2009.

3 Work Plan

3.1 Activities

1. Start up seminar: two weeks at Unicamp, introduction to development tools and the use of the tools for recording classes. First videos productions and analysis.
2. Weekly recording of video.
3. Weekly meeting: 4 hours of weekly discussion of future subjects and analysis and correction of recorded videos
4. Summer seminar: two weeks at Unicamp, studying the needed mathematics of the upcoming year.
5. Winter seminar: one week at Unicamp, intensive work on video and studying mathematics

3.2 Schedule

Activity	Months 1 to 3	4 to 6	7 to 9	10 to 12
Start up seminar	X			
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X
Summer seminar			X	X
Winter seminar		X		

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Statistic and Probability 2 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

This scholarship is intended for deaf students in the STEM field. He will be part of a group that will record in Libras the answers to more than 1400 ENEM mathematics questions. He will be one of four students focusing on number theory and basic arithmetic operations. This program is a consequence and continuation of a pilot project run at Unicamp in 2023.

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3. Create the first content of a website that will be the main reference for studying mathematics in Libras.
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This will be achieved by recording in Libras the answer to each of the mathematics questions of ENEM - Brazilian National High School Exam. There are more than 1400 questions considering all the editions since 2009.

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1. Start up seminar: two weeks at Unicamp, introduction to development tools and the use of the tools for recording classes. First videos productions and analysis.
2. Weekly recording of video.
3. Weekly meeting: 4 hours of weekly discussion of future subjects and analysis and correction of recorded videos
4. Summer seminar: two weeks at Unicamp, studying the needed mathematics of the upcoming year.
5. Winter seminar: one week at Unicamp, intensive work on video and studying mathematics

3.2 Schedule

Activity	Months 1' to 3'	4' to 6'	7' to 9'	10' to 12'
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Statistic and Probability 3 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

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2. Weekly recording of video.
3. Weekly meeting: 4 hours of weekly discussion of future subjects and analysis and correction of recorded videos
4. Summer seminar: two weeks at Unicamp, studying the needed mathematics of the upcoming year.
5. Winter seminar: one week at Unicamp, intensive work on video and studying mathematics

3.2 Schedule

Activity	Months 1 to 3	4 to 6	7 to 9	10 to 12
Start up seminar	X			
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X
Summer seminar			X	X
Winter seminar		X		

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Statistic and Probability 3 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

This scholarship is intended for deaf students in the STEM field. He will be part of a group that will record in Libras the answers to more than 1400 ENEM mathematics questions. He will be one of four students focusing on number theory and basic arithmetic operations. This program is a consequence and continuation of a pilot project run at Unicamp in 2023.

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3. Create the first content of a website that will be the main reference for studying mathematics in Libras.

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4. Summer seminar: two weeks at Unicamp, studying the needed mathematics of the upcoming year.
5. Winter seminar: one week at Unicamp, intensive work on video and studying mathematics

3.2 Schedule

Activity	Months 1' to 3'	4' to 6'	7' to 9'	10' to 12'
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Statistic and Probability 4 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

This scholarship is intended for deaf students in the STEM field. He will be part of a group that will record in Libras the answers to more than 1400 ENEM mathematics questions. He will be one of four students focusing on number theory and basic arithmetic operations. This program is a consequence and continuation of a pilot project run at Unicamp in 2023.

2 Objectives

1. Promote social inclusion of deaf people in Brazil
2. Develop the vocabulary of mathematics in Brazilian Sign Language - Libras
3. Create the first content of a website that will be the main reference for studying mathematics in Libras.
4. Develop a community of deaf people engaged in learning mathematics

This will be achieved by recording in Libras the answer to each of the mathematics questions of ENEM - Brazilian National High School Exam. There are more than 1400 questions considering all the editions since 2009.

3 Work Plan

3.1 Activities

1. Start up seminar: two weeks at Unicamp, introduction to development tools and the use of the tools for recording classes. First videos productions and analysis.
2. Weekly recording of video.
3. Weekly meeting: 4 hours of weekly discussion of future subjects and analysis and correction of recorded videos
4. Summer seminar: two weeks at Unicamp, studying the needed mathematics of the upcoming year.
5. Winter seminar: one week at Unicamp, intensive work on video and studying mathematics

3.2 Schedule

Activity	Months 1 to 3	4 to 6	7 to 9	10 to 12
Start up seminar	X			
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X
Summer seminar			X	X
Winter seminar		X		

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Statistic and Probability 4 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

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3.2 Schedule

Activity	Months 1' to 3'	4' to 6'	7' to 9'	10' to 12'
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Numbers and basic operations 1 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
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Start up seminar	X			
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X
Summer seminar			X	X
Winter seminar		X		

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[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Numbers and basic operations 1 - PART B

Principal Investigator: *Marcelo Firer*
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Vertical(s): Innovation
Line(s) of research:
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CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Numbers and basic operations 2 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
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Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X
Summer seminar			X	X
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CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Numbers and basic operations 2 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
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CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Numbers and basic operations 3 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
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CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Numbers and basic operations 3 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
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CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Numbers and basic operations 4 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
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CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Geometry 1 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

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References

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CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Geometry 1 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
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CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Geometry 2 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
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Start up seminar	X			
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X
Summer seminar			X	X
Winter seminar		X		

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CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Geometry 2 - PART B

Principal Investigator: *Marcelo Firer*
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Vertical(s): Innovation
Line(s) of research:
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CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Geometry 3 - PART A

Principal Investigator: *Marcelo Firer*
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Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
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Mathematics vocabulary in Brazilian Sign Language - Libras
Geometry 3 - PART B

Principal Investigator: *Marcelo Firer*
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CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Geometry 4 - PART A

Principal Investigator: *Marcelo Firer*
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Line(s) of research:
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Mathematics vocabulary in Brazilian Sign Language - Libras
Geometry 4 - PART B

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

Vertical(s): Innovation
Line(s) of research:
Curricular Innovations, Impact, and Outreach
Mathematics Education and Teacher Training

1 Summary

This scholarship is intended for deaf students in the STEM field. He will be part of a group that will record in Libras the answers to more than 1400 ENEM mathematics questions. He will be one of four students focusing on number theory and basic arithmetic operations. This program is a consequence and continuation of a pilot project run at Unicamp in 2023.

2 Objectives

1. Promote social inclusion of deaf people in Brazil
2. Develop the vocabulary of mathematics in Brazilian Sign Language - Libras
3. Create the first content of a website that will be the main reference for studying mathematics in Libras.
4. Develop a community of deaf people engaged in learning mathematics

This will be achieved by recording in Libras the answer to each of the mathematics questions of ENEM - Brazilian National High School Exam. There are more than 1400 questions considering all the editions since 2009.

3 Work Plan

3.1 Activities

1. Start up seminar: two weeks at Unicamp, introduction to development tools and the use of the tools for recording classes. First videos productions and analysis.
2. Weekly recording of video.
3. Weekly meeting: 4 hours of weekly discussion of future subjects and analysis and correction of recorded videos
4. Summer seminar: two weeks at Unicamp, studying the needed mathematics of the upcoming year.
5. Winter seminar: one week at Unicamp, intensive work on video and studying mathematics

3.2 Schedule

Activity	Months 1' to 3'	4' to 6'	7' to 9'	10' to 12'
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Algebra 1 - PART A

Principal Investigator: *Marcelo Firer*
Co-PI: *Leonardo Barichello*

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Activity	Months 1 to 3	4 to 6	7 to 9	10 to 12
Start up seminar	X			
Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X
Summer seminar			X	X
Winter seminar		X		

References

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CBG Scientific Initiation Fellowship Proposal

Mathematics vocabulary in Brazilian Sign Language - Libras
Algebra 1 - PART B

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Weekly recording of video	X	X	X	X
Weekly meeting	X	X	X	X

References

[1] Libras + Mat, https://www.ime.unicamp.br/~libras

CBG Scientific Initiation Fellowship Proposal

Geometry of Numbers: Minkowski's theorem and Algebraic Integers

Principal Investigator: *Ethan Cotteril*
Co-PI: *Tiago Jardim da Fonseca*

Vertical: Algebraic Geometry
Line of action: Algebra and Arithmetic

1 Summary

Consider the list of the first ten prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. Among these, some can be written as the sum of two squares:

$$2 = 1^1 + 1^2,\ 5 = 2^2 + 1^2,\ 13 = 3^2 + 2^2,\ 17 = 4^2 + 1^2,\ 29 = 5^2 + 2^2,$$

All of them, except the prime 2, are congruent to 1 modulo 4. In fact, one can check that such a congruence is a necessary condition for an odd prime number to be written as a sum of two squares. Surprisingly, the converse is also true:

Theorem 1.1. *If p is a prime number such that $p \equiv 1 \pmod 4$, then there are integers a and b such that $p = a^2 + b^2$.*

The true explanation for this phenomenon lies in the study of the arithmetic of the ring of *Gaussian integers*

$$\mathbb{Z}[i] = \{a + ib \in \mathbb{C} : a, b \in \mathbb{Z}\}.$$

A prime p which can be written as a sum of two squares factors in $\mathbb{Z}[i]$:

$$p = a^2 + b^2 = (a + ib)(a - ib).$$

The above theorem amounts to proving that a prime $p \in \mathbb{Z}$ satisfying $p \equiv 1 \pmod 4$ is not a prime element of the ring $\mathbb{Z}[i]$.

The ring $\mathbb{Z}[i]$ is one of the simplest examples of a *ring of algebraic integers*. *Algebraic Number Theory* is the area of Mathematics devoted to the study of the arithmetic of these more complicated rings. One of its central problems is to understand when unique factorization holds or fails; for instance, $\mathbb{Z}[i\sqrt{5}]$ is not a unique factorization domain, since $6 = 2 \cdot 3 = (1 + i\sqrt{5})(1 - i\sqrt{5})$. However, Dedekind showed that these rings still satisfy a remarkable unique factorization property in terms of prime ideals. This is the starting point to the study of more sophisticated invariants of a ring of algebraic integers R, such as its *class group* (or *Picard group*) $Cl(R)$.

One of the main results of Algebraic Number Theory is that the class group $Cl(R)$ is always finite. The proof of this theorem is *geometric*; it relies in an essential way on the following theorem of Minkowski, which has many other applications in Number Theory and other areas of Mathematics.

Theorem 1.2 (Minkowski)**.** *Let L be a lattice in $\mathbb{R}^n$ with fundamental domain F, and let X be a convex centrally symmetric body in $\mathbb{R}^n$. If*

$$vol(X) > 2^n vol(F)$$

then X contains a non-zero element of L.

2 Objectives

This project is an introduction to geometric methods in Number Theory, with applications to the theory of Algebraic Integers. Its main objectives comprise an algebraic part, concerning the study of rings of algebraic integers (including the theorem of Dedekind on the unique factorization in prime ideals), and a geometric part, centered in understanding how to apply Minkowski's 'geometry of numbers' to algebraic integers.

3 Methods

We shall mainly follow the methodology in parts I and II of [3], using [2] and [1] as supporting references. We also expect to work out in detail explicit examples in the case of quadratic and cyclotomic integers.

4 Schedule

1. Review some basic algebraic background (such as rings, field extensions, modules) and study algebraic numbers and rings of algebraic integers, including a careful analysis of the case of quadratic and cyclotomic integers. Main reference: [3, Chapters 1, 2, 3].

2. Study prime ideals and Dedekind's unique factorization theorem. Definition of the class group. Main reference: [3, Chapters 4, 5, 9].

3. Study lattices (and their quotient tori) and Minkowski's theorem. Work out explicit examples and applications for the two-square and four-square theorems. Main reference: [3, Chapters 6, 7].

4. Apply Minkowski's theorem to the finiteness of the class group, and study some applications such as Kummer's cases of Fermat's Last Theorem. Main reference: [3, Chapters 8, 9, 11].

Activity	Quarter 1	Quarter 2	Quarter 3	Quarter 4
1	X			
2		X		
3			X	
4				X

References

[1] J. Neukirch, *Algebraic number theory.* Translated from the 1992 German original and with a note by Norbert Schappacher. With a foreword by G. Harder. Grundlehren der Mathematischen Wissenschaften, 322. Springer-Verlag, Berlin, 1999. 1

[2] P. Samuel, *Algebraic theory of numbers.* Translated from the French by Allan J. Silberger Houghton Mifflin Co., Boston, Mass. 1970 1

[3] I. Stewart, D. Tall, *Algebraic number theory and Fermat's last theorem.* Fourth edition. CRC Press, Boca Raton, FL, 2016. 1, 2

CBG Scientific Initiation Fellowship Proposal

The Mordell–Weil theorem and 2-divisible points on elliptic curves

Principal Investigator: *Ethan Cotteril*
Co-PI: *Tiago Jardim da Fonseca*

Vertical: Algebraic Geometry
Line of action: Algebra and Arithmetic

1 Summary

Let k be a field of characteristic different from 2. Recall that an *elliptic curve* E over k is a projective algebraic curve which admits an equation of the form

$$E : y^2z = z^3 f(x/z),$$

where $f(x) \in k[x]$ is a separable polynomial of degree 3. One of the distinguinshing features of elliptic curves comes from the fact that they admit a natural (commutative) *group structure* given by algebraic formulae — that is, an elliptic curve have the structure of an *algebraic group*, an algebraic analogue of the notion of Lie group in Differential Geometry. This is arguably the reason why elliptic curves play a central role in many developments in Algebraic Geometry, Number Theory, and some areas of Applied Mathematics.

It follows from the fact that an elliptic curve E over k is a commutative algebraic group that its set of k-rational points $E(k)$ is an abelian group. The following theorem is one of the cornerstones of Arithmetic Algebraic Geometry.

Theorem 1.1 (Mordell–Weil)**.** *If k is a number field, then the abelian group $E(k)$ is finitely generated.*

The first step in the proof of the above theorem is to prove the so-called 'weak Mordell–Weil theorem', which asserts that the quotient $E(k)/2E(k)$ is finite. Here, one must understand the subgroup of 2-divisible elements $2E(k)$. For instance, a classic result asserts that, when splits completely in $k[x]$, say

$$f(x) = (x - r_1)(x - r_2)(x - r_3), \qquad r_i \in k,$$

a point $P = [a : b : 1] \in E(k)$ is 2-divisible if and only if $a - r_i$ is a square in k for every $i \in \{1, 2, 3\}$. The traditional proofs of this statement rely either on heavy computation or on sophisticated technology such as local fields and Galois cohomology. A recent work of Bekker and Zarhin [1] gives a new, simpler, proof of this result. Their method yields new applications to the description of elliptic curves with a given torsion subgroup and is also related to similar problems for Jacobians of higher genus curves.

2 Objectives

This project is an introduction to the theory of rational points on elliptic curves. Its main objectives are to understand in detail a proof of the celebrated Mordell–Weil theorem and to study the so-called '2-descent' via the recent methods of Bekker and Zarhin.

3 Methods

There are many classic references for the arithmetic theory of elliptic curves; we shall mainly follow the methodology of [3] and [2] for the basic material. For the 2-descent and characterisation of 2-divisible points, we shall follow the approach of the paper [1].

4 Schedule

1. Review some basic algebraic and geometric background, such as plane algebraic curves, homogeneous polynomials, and the projective space. Main references: [2, Sections I.1, I.2] and [3, Appendix A].

2. Study the definition of elliptic curves, Weierstrass equations, their group structure, and points of finite order. Main references: [2, Sections I.3, I.4, II.1, II.2, and II.5] and [3, Chapters 1 and 2].

3. Study 2-divisibility on elliptic curves and a proof of the weak Mordell–Weil theorem. Main references [3, Chapter 3] and [1].

4. Study heights and complete the proof of the Mordell–Weil theorem. If time permits, study elliptic curves over finite fields, and applications of the methods of [1] to existence problems. Main reference: [3, Chapter 3] and [1].

Activity	Quarter 1	Quarter 2	Quarter 3	Quarter 4
1	X			
2		X		
3			X	X
4				X

References

[1] B. M. Bekker, Y. G. Zarhin, *Division by 2 of rational points on elliptic curves.* St. Petersbg. Math. J. 29, No. 4, 683-713 (2018). 1, 2

[2] J. S. Milne, *Elliptic curves.* BookSurge Publishers, Charleston, SC, 2006. 1

[3] J. S. Silverman, J. T. Tate, *Rational Points on Elliptic Curves.* Second Edition. Springer Cham Heidelberg New York Dordrecht London, 2015. 1, 2

CBG Scientific Initiation Fellowship Proposal

Geometric Invariant Theory applied to the classification of curves

PI: *Marcos Jardim*
Thematic Vertical: *Algebraic Geometry*
Line of action: *Moduli Spaces*

Summary

Geometric Invariant Theory (GIT), developed by Mumford around the 1960's , applies the ideas from Classical Invariant theory to solve problems from Algebraic Geometry. An important problem both in Classical and Geometric Invariant Theory, known as Hilbert's 14^{th} problem, was to determine when the algebra of invariants was finitely generated. Furthermore, GIT acts as an important tool in the construction of quotients, this method applied to classification problems will be our primary focus on this project.

A classification problem involves studying a collection of objects X up to some equivalence relation $\sim$. A standard way to approach this problem is to study the orbit space $X/G := \{G\cdot x \mid x \in X\}$ of a group G acting on a topological space X, and equipping X/G with the quotient topology. However X/G often lacks desirable geometric properties that in some sense represent the underlying geometry.

In this project, we work with an affine algebraic group acting on a variety, or more generally, a scheme. Essentially, an affine algebraic group is an affine scheme with a group structure such that multiplication and inversion are morphisms of schemes. In such cases, the orbit space doesn't usually posses a scheme structure. GIT provides a method for constructing quotients with desirable geometric properties, yet imposes some restrictions, such as working with reductive groups and obtaining a quotient not for the whole scheme, but for a subset of so called "stable points"

Throughout our study, we will be interested in the specific scenario where X denotes the set of plane curves of degree d and $GL(3,k)$ acts on X, with k being an algebraic closed field with characteristic zero. An algebraic plane curve is an homogeneous ideal in the polynomial ring $k[x, y, z]$, thus, the set of all homogeneous polynomials of degree d gives a vector space S_d, and the associate projective space $\mathbb{P}(S_d)$ yields the set of all algebraic plane curves of degree d.

Despite $\mathbb{P}(S_d)$ and $GL(3,k)$ being schemes, the orbit space $\mathbb{P}(S_d)/GL(3,k)$ is not a scheme. In this setting, GIT introduces the concept of stability that in some sense "filters" the desirable orbits in order to obtain a quotient with scheme structure.

Objectives

The primary objective of this project is to study Geometric Invariant Theory as tool to solve classification problems, focusing specifically on the classification of plane cubics. To achieve this goal, it is essential to first grasp the fundamentals from Scheme Theory, which we consider a secondary objective.

Methodology

To achieve our objectives, we will follow the methods employed in references [1] and [2]. Additionally, we will apply this methods through examples to enhance our understanding.

Schedule

1. Study the fundamentals of schemes, following the chapters I, II, III from [3].
2. Study algebraic groups and understand the concepts of categorical and good quotients following chapter III from reference [1].
3. Work on some examples from [1] and study the GIT Affine Quotient following [1] and [2].
4. Study projective schemes using chapter IX from [3] and projective Geometric Invariant Theory as discussed in references [1] and [2] .
5. Understand the Hilbert-Mumford Criterion using [2] and study the classification of plane cubics and other examples as proposed [1] and [2].

Activity	**Quarter 1**	**Quarter 2**	**Quarter 3**	**Quarter 4**
1	X	X		
2		X		
3			X	
4			X	X
5				X

References

[1] V. Hoskins, *Moduli Problems and Geometric Invariant Theory*, lecture notes.

[2] P.E. Newstead, *Introduction to Moduli Problems and Orbit Spaces*, Tata Institute of Fundamental Research, 1978.

[3] G. Ellingsrud and J. C. Ottem, *Introduction to schemes*, version 2.2, University of Oslo, 2022.

CBG Scientific Initiation Fellowship Proposal

Categories and applications in Algebraic Geometry and Algebraic Topology

PI: *Marcos Jardim*
Thematic Vertical: *Algebraic Geometry*
Line of action: *Moduli spaces*

Summary

In the middle of the twentieth century, the use of algebra in topology was expanding and lots of results indicated that homology groups would provide an algebraic vision to topology. In this context, while Samuel Eilenberg and Saunders Mac Lane were studying the connection between homology and cohomology groups, they realized that a tool formalizing the transition between algebra and topology was necessary, so they started working on what we now call category theory.

But category theory provides more than that, it is a general theory of mathematical structures and its relations. It allows us to connect different areas of formal sciences and formalize models in math, logic, physics and computer science. Lots of constructions of new mathematical objects from another ones are conveniently expressed and unified in terms of category theory, such as quotient spaces, direct products, completion and duality.

A category is, basically, a collection of objects and transformations between them, the morphisms. An advantage this packaging process brings is the existence of a general categorical definition of isomorphisms, that can be restricted into the definitions of group isomorphisms, topological spaces homeomorphisms and even categories isomorphisms.

Besides the already mentioned relation between algebra and topology, math is full of constructions that translate a mathematical object into another. This way, it becomes interesting to study functors. A functor can describe an equivalence of categories; in this case, objects of a given category can be translated into or reconstructed from objects of another category. A simple example in linear algebra is the equivalence between the category of finite dimensional vector spaces and the category in which objects are natural numbers and morphisms are matrices.

This project focuses in studying the structure of this important tool and some interesting applications in algebraic topology and algebraic geometry. When it comes to algebraic topology, we can mention the fundamental group, the Van Kampen theorem and homology and cohomology groups. Related to algebraic geometry, we are going to study affine varieties, projective varieties and sheaves.

Objectives

The main objective of this project is to study category theory and understand how its concepts relate to algebraic topology and algebraic geometry.

In order to get a satisfactory comprehension of the applications, it will be necessary to study concepts of both areas. Thus, this would be a parallel goal of the project.

Finally, given the lack of written material in Portuguese about the theme, the project also aims the production of an introductory text about category theory in Portuguese.

Methodology

This project will employ analytical methods to explore the research questions. Such theoretical work will be developed following the references listed at the end of this project and the topics presented will be weekly discussed with the advisor.

Schedule

1. Complete graduate courses.
2. Study first three chapters of [1].
3. Study fundamental group and homology following [6], cohomology following [3] and the Van Kampen Theorem following [4] and [6].
4. Study Corollary 3.8, Theorem 4.4 and Proposition 4.10 of [3].
5. Study chapters IV, V and VI of [1].
6. Study direct and inverse images of sheaves following [5] and [3].
7. Present on UNICAMP's scientific initiation conference.

Activity	**Quarter 1**	**Quarter 2**	**Quarter 3**	**Quarter 4**
1	X	X	X	X
2	X			
3		X		
4		X		
5			X	
6				X
7			X	

References

[1] Saunders Mac Lane. *Categories for the Working Mathematician.* New York, Springer, 2010.

[2] Riehl, Emily. *Category Theory in Context.* Courier Dover Publications, 9 Mar. 2017.

[3] Hartshorne, Robin. *Algebraic Geometry.* New York, Springer, 1977.

[4] J Peter May, and University Of Chicago Press. *A Concise Course in Algebraic Topology.* Chicago ; London, University Of Chicago Press, [Ca], Cop, 2016.

[5] Ellingsrud, Geir, and John Christian Ottem. *Introduction to Schemes.* University of Oslo, 29 Aug. 2022.

[6] Hatcher, Allen. *Algebraic Topology.* New York, Cambridge University Press, 2002.

CBG Scientific Initiation Fellowship Proposal

Introduction to Algebraic Varieties

PI: *Marcos Jardim*
Thematic Vertical: *Algebraic Geometry*
Line of action: *line*

Summary

Algebraic varieties are of extreme importance, as they form the basis of algebraic geometry. Varieties are solutions to systems of polynomial equations. In this project, we aim to provide an introduction to algebraic varieties in the affine and projective cases.

Consider k an algebraically closed field. Let's define the n-affine space over k (denoted by $\mathbb{A}_k^n$ or $\mathbb{A}^n$) formed by the n-tuples with entries in the field k. This way,

$$\mathbb{A}^n = \{(a_1, a_2, \ldots, a_n)|\ a_1, a_2, \ldots, a_n \in k\}$$

Now, consider the polynomial ring $k[x_1, x_2, \ldots, x_n]$ and let $T \subset k[x_1, x_2, \ldots, x_n]$. Let

$$Z(T) = \{P \in \mathbb{A}^n|\ f(P) = 0,\ \forall\ f \in T\}$$

A subset $A \subseteq \mathbb{A}^n$ is said to be affine algebraic if there exists $T \subset k[x_1, x_2, \ldots, x_n]$ such that $A = Z(T)$.

Given this definition, we can define a topology on $\mathbb{A}^n$ called the Zariski topology, where the closed sets are the affine algebraic sets. We define an affine algebraic variety as a closed and irreducible subset of $\mathbb{A}^n$ with the Zariski topology.

The n-projective space over k, denoted by $\mathbb{P}_k^n$ or $\mathbb{P}^n$, is formed by the $(n+1)$-tuples with an equivalence relation.

$$\mathbb{P}^n = \{(a_0 : a_1 : \cdots : a_n)|\ a_0, a_1, \ldots, a_n \in k\} / \sim$$

where

$$(a_0 : a_1 : \cdots : a_n) \sim (b_0 : b_1 : \cdots : b_n) \Longleftrightarrow (a_0, a_1, \ldots, a_n) = \lambda(b_0, b_1, \ldots, b_n)$$

where $\lambda \in k^*$ and the equality occurs in $\mathbb{A}^{n+1}$. We need the polynomials to be homogeneous so that the zeros are well-defined, i.e., they do not depend on the representative. We say that $f \in k[x_0, x_1, \ldots, x_n]$ is homogeneous if $f(\lambda a_0, \lambda a_1, \ldots, \lambda a_n) = \lambda^d f(a_0, a_1, \ldots, a_n)$ for each $\lambda \in k^*$ and $\deg(f) = d$.

Now, we just follow the same steps as in the affine case, but with homogeneous elements. Take a subset $T \subset k[x_0, x_1, \ldots, x_n]$ consisting of homogeneous polynomials, and thus

$$Z(T) = \{P \in \mathbb{P}^n |\ f(P) = 0,\ \forall\ f \in T\}$$

A subset $B \subseteq \mathbb{P}^n$ is said to be projective algebraic if there exists a subset $T \subseteq k[x_0, x_1, \ldots, x_n]$ consisting of homogeneous polynomials such that $B = Z(T)$. Also, we have the Zariski topology where closed sets are projective algebraic sets. Finally, we define a projective algebraic variety as an irreducible projective algebraic set.

With the definitions of algebraic varieties, we want to study the functions defined on them, which will be the regular functions and will form a ring called the ring of regular functions. Such a ring can provide some geometric properties of the variety, for example, if the local ring is regular at a point of the variety, then the point is non-singular.

Objectives

In this project, we aim to conduct a comprehensive study of algebraic varieties, as they form the foundation of algebraic geometry. Starting from varieties, we will demonstrate the main results of classical algebraic geometry.

As our goal is to study algebraic varieties, we need to consider explicit examples of varieties. Some examples of varieties are:

1. Hypersurfaces
2. Twisted cubic
3. Complete intersection
4. Grassmannian
5. Veronese Variety

So, we will study singularities, rational functions, and morphisms between the varieties mentioned above.

Methodology

For this project, we will use the first chapter of [1], which is one of the primary and most renowned texts on algebraic geometry. To fully understand the book [1], we will need results from commutative algebra, and for that, we will use the reference [2]. We will also use the references [3] and [4] to study varieties.

References

[1] R. Hartshorne, *Algebraic geometry*. Graduate Texts in Mathematics. Springer New York, 2013.

[2] M. Atiyah, *Introduction to commutative algebra*. CRC Press, 2018.

[3] A. Gathmann, *Algebraic geometry*. Notes for a class available at https://agag-gathmann.math.rptu.de/class/alggeom-2021/alggeom-2021.pdf, 2023.

[4] R. Shafarevich, *Basic algebraic geometry*. Springer, 1994.

CBG Scientific Initiation Fellowship Proposal

Derived Categories and Fourier-Mukai Transforms

PI: *Marcos Jardim*
Thematic Vertical: *Algebraic Geometry*
Line of action: *Moduli spaces*

Summary

In this project, we will study derived categories and Fourier-Mukai transforms with the aim of understanding the construction of these objects and their main tools. Special focus will be given to the construction of derived categories of coherent sheaves on a scheme and their algebraic properties. As the main application of the methods studied, the project will culminate in describing Fourier-Mukai transforms between an abelian variety and its dual variety, following Shigeru Mukai's original work. The tools studied here could also be used in future research in Algebraic Geometry and Gauge Theory, especially in topics such as Moduli Spaces, Bridgeland stability and instanton sheaves.

Introduction

The development of modern Algebraic Geometry has historically been closely intertwined with the development of Category Theory, which, particularly due to the works of A. Grothendieck and J. Verdier in the mid-1960s, became a fundamental part of structuring current problems in the field. Among the tools obtained through categories, **derived categories** have proven to be highly valuable in the study of geometric objects today: from their ability to simplify the definition and manipulation of derived functors, for example, important for studying the cohomology of sheaves on an algebraic variety; to their ability to reframe geometric problems in entirely categorical terms.

In general, given an abelian category $\mathcal{M}$, its derived category $D(\mathcal{M})$ is the localization of the category of chain complexes $\mathrm{Kom}(\mathcal{M})$ with objects in $\mathcal{M}$ with respect to quasi-isomorphisms, i.e., to a class of morphisms $M^\bullet \to N^\bullet$ of chain complexes in $\mathcal{M}$ that preserve their cohomology groups unchanged. The formalization of this idea involves the construction of differential graded categories and triangulated categories, and allows for the definition of derived functors on the right and left, given the existence of enough injectives, projectives, and/or flats in $\mathcal{M}$, depending on the case.

There are several techniques for manipulating and studying derived categories. In this work, we will focus on understanding **Fourier-Mukai transforms**, integral functors first introduced by Mukai [3] in 1981, inspired by Fourier's work: given X and Y projective varieties, an integral functor $\phi^{\mathcal{K}^\bullet}_{X\to Y} : \mathcal{D}(X) \to \mathcal{D}(Y)$ between their derived categories of coherent sheaves is of the form

$$\phi^{\mathcal{K}^\bullet}_{X\to Y}(\mathcal{E}^\bullet) = \mathbf{R}\pi_{Y*}(\pi_X^*\mathcal{E}^\bullet \overset{\mathbf{L}}{\otimes} \mathcal{K}^\bullet),$$

where π_X, π_Y are the canonical projections onto the factors of $X \times Y$, $\mathcal{K}^\bullet \in \mathcal{D}(X \times Y)$ is the so-called *kernel* of the transform, and $\mathbf{R}$ and $\mathbf{L}$ denote right and left derived functors, respectively. In the case where this functor is an equivalence between derived categories and $\mathcal{K}^\bullet$ is concentrated, it is called a *Fourier-Mukai transform.*

In addition to definitions and main properties, applications of derived categories and Fourier-Mukai transforms in Algebraic Geometry will also be studied, especially over abelian varieties. In particular, we seek to understand the results obtained in S. Mukai's article *Duality between $D(X)$ and $D(\hat{X})$ with Its Application to Picard Sheaves* [3], which relates the derived category of coherent sheaves of an abelian variety to that of its dual abelian variety. The case where X has dimension 1, corresponding to a nonsingular elliptic curve, will also be studied in depth.

Alongside this main reference, the following books will be used: *Derived categories* [5] by A. Yekutiely, for the study of properties of derived categories and functors; *Fourier-Mukai Transforms in Algebraic Geometry* [2] by D. Huybrechts, for the study of derived categories of coherent sheaves, available in Chapter 3; *Abelian Varieties* [4] by D. Mumford, for the study of abelian varieties and their properties; and *Fourier-Mukai and Nahm transforms in geometry and mathematical physics* [1] by U. Bruzzo, C. Bartocci, and D. Hernández-Ruipérez, primarily Chapters 1, 2, and 3, as auxiliary references while reading Mukai's article.

Objectives

The objective of this project is to learn to use the tools of derived categories and Fourier-Mukai transforms in the study of objects arising from Algebraic Geometry, especially the derived category of coherent sheaves on an abelian variety and its geometric properties.

The techniques developed here can be subsequently used in research on Gauge Theory and Moduli Spaces, such as studies related to Bridgeland Stability and instanton sheaves.

Schedule

The research aims to understand the results in article [3], by S. Mukai, on the construction of Fourier-Mukai functors between derived categories of coherent sheaves and their connection to the structure of Picard sheaves on abelian varieties. To achieve this, the study will be divided into three parts, spanning from March to December of 2023.

In the first part, we will study the more general categorical structures of abelian categories, differential graded categories, triangulated categories, and triangulated functors, as well as the localization of categories; finally, we will explore the structure of derived

Table 1: Project Schedule

		Month								
Project Part and Description		1st	2nd	3rd	4th	5th	6th	7th	8th	9th
First Part	Derived categories and prerequisites	X	X	X						
Second Part	Derived category of coherent sheaves				X	X				
Third Part	Fourier-Mukai transforms on abelian varieties						X	X	X	X

categories and derived functors, along with their main properties. These topics are covered between Chapters 2 and 9 of [5]. As an application and important outcome of this initial study, the second part of the project will describe the structure of the derived category of coherent sheaves on a scheme, available in Chapter 3 of [2].

Finally, in the third part, we will study S. Mukai's article [3], covering the definition and properties of Fourier-Mukai transforms, the correspondence between derived categories of coherent sheaves of an abelian variety and its dual variety, and the use of these results in the study of Picard sheaves on these varieties. Special attention will be given to the case where the abelian variety has dimension 1, also corresponding to a nonsingular elliptic curve, whose Picard sheaf structure can be described in greater detail.

References

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[5] Amnon Yekutiely. *Derived Categories*. Cambridge Studies in Advanced Mathematics 183. Cambridge ; New York, NY: Cambridge University Press, 2020. ISBN: 978-1-108-29282-5.

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